Origami : Local Foldability

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KAIST

December 21, 2007

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Brief Example

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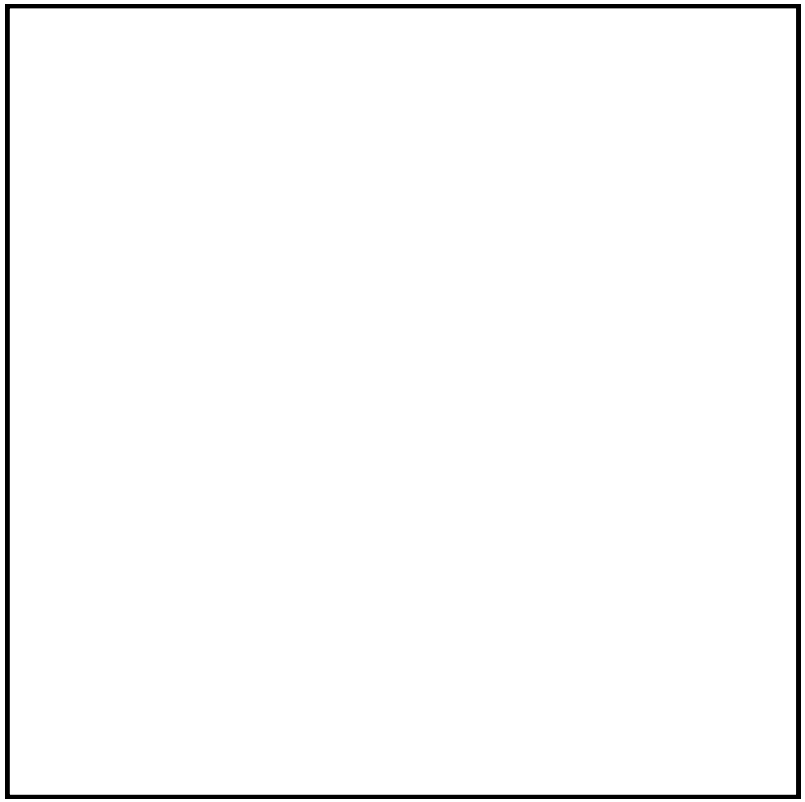
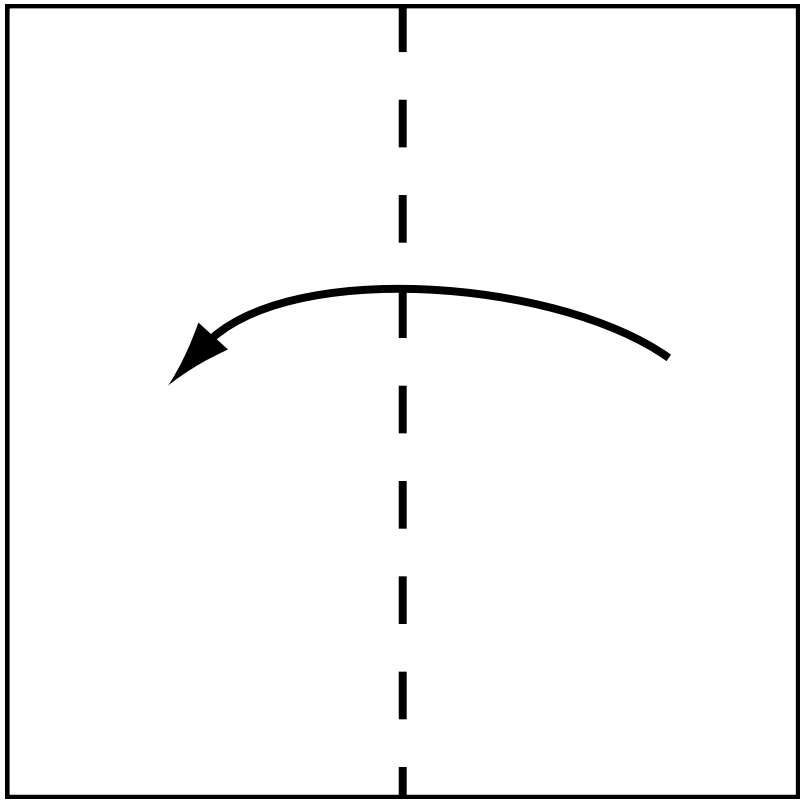


Brief Example



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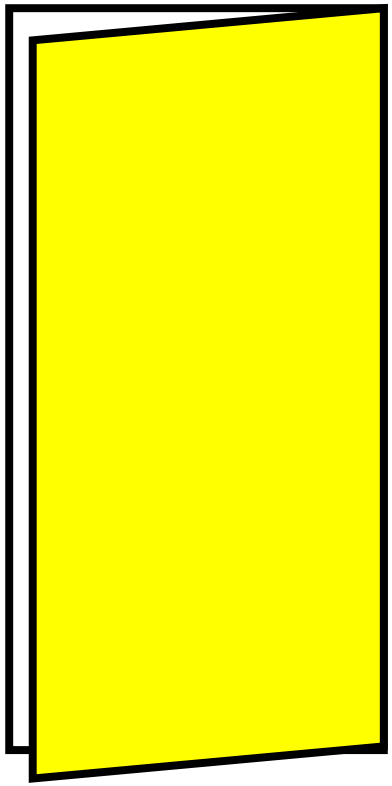

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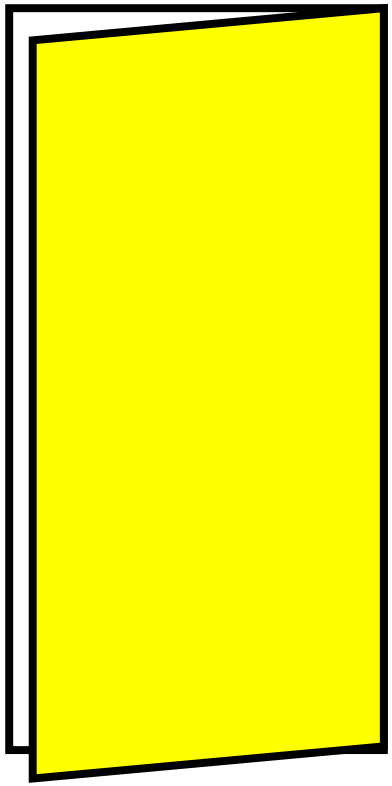
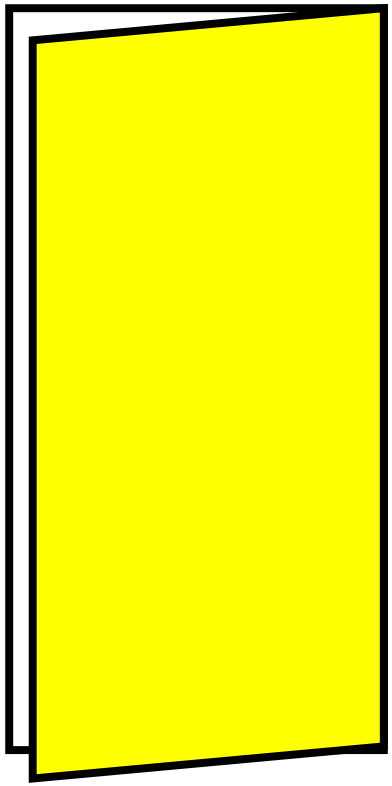


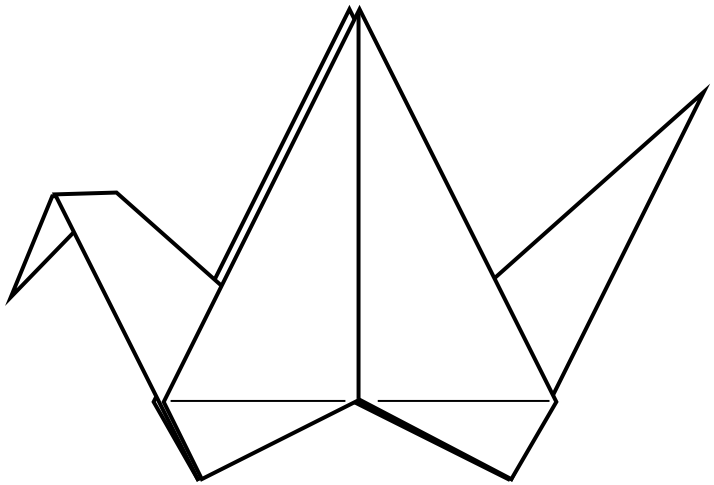
This is origami!

Brief Example



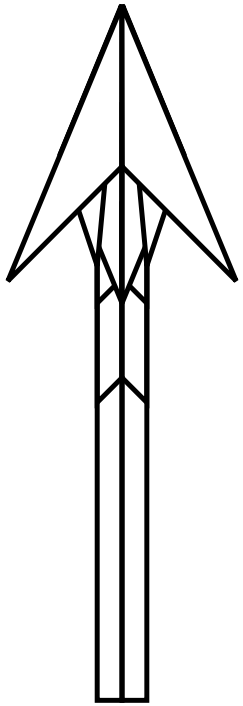
This is origami! If you want to do more further....

Brief Example



Crane

Brief Example



Arrow

Brief Example

Sailing Ship

Brief Example

Sea Urchin

Brief Example

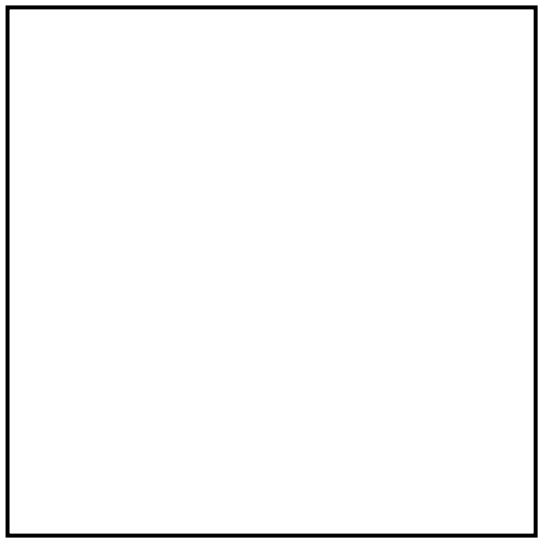
Girl

Brief Example

Housemaid, by Kamiya Ryo

Goal of Origami

Goal of Origami



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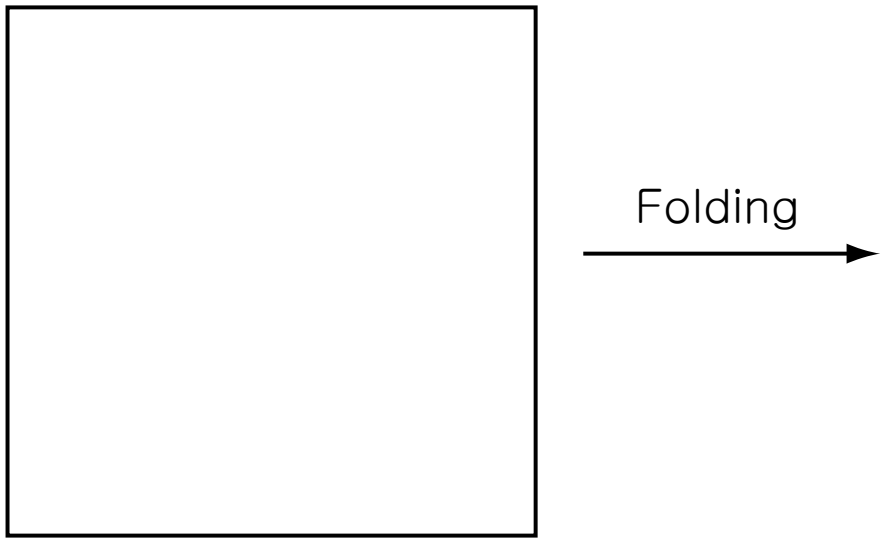

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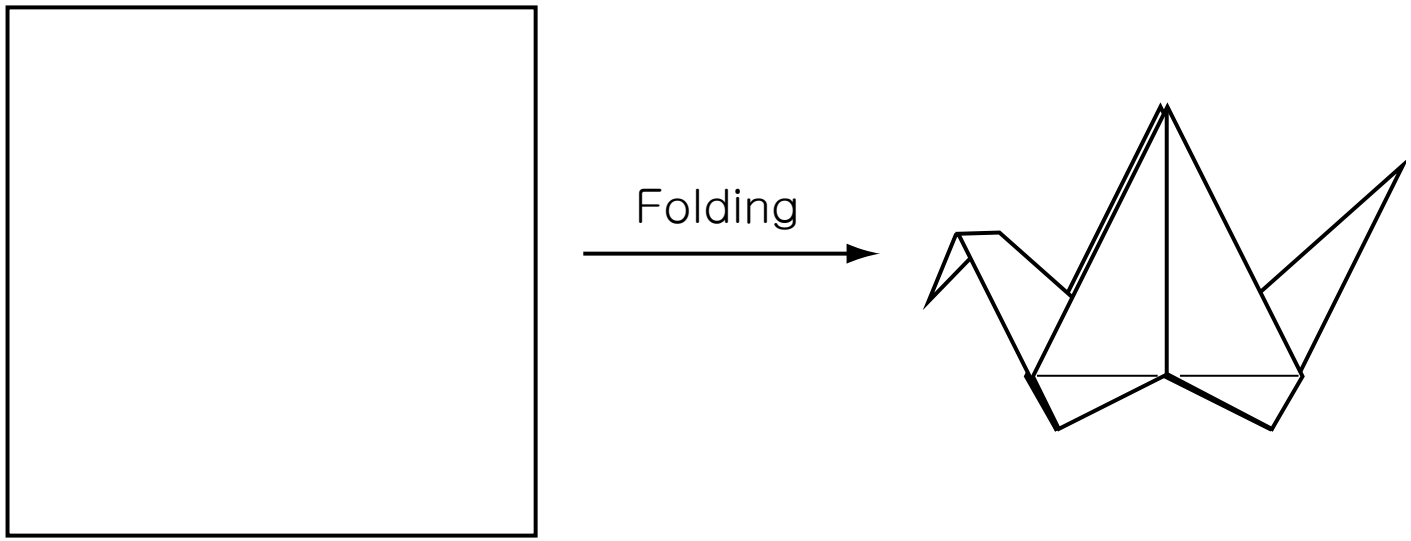

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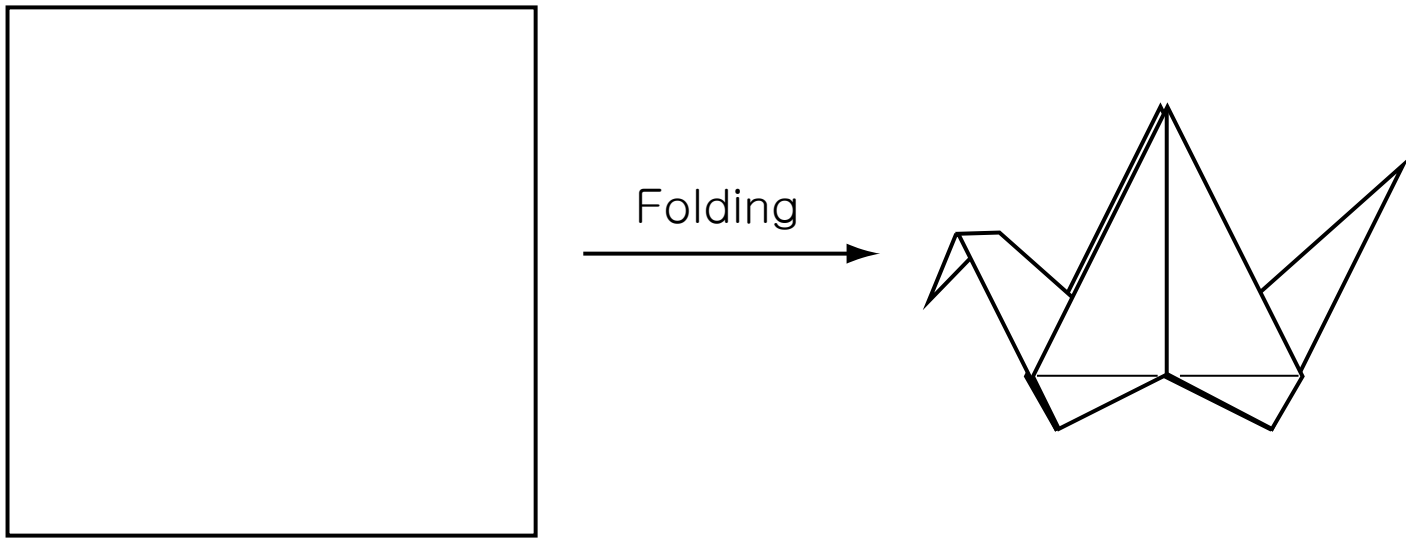
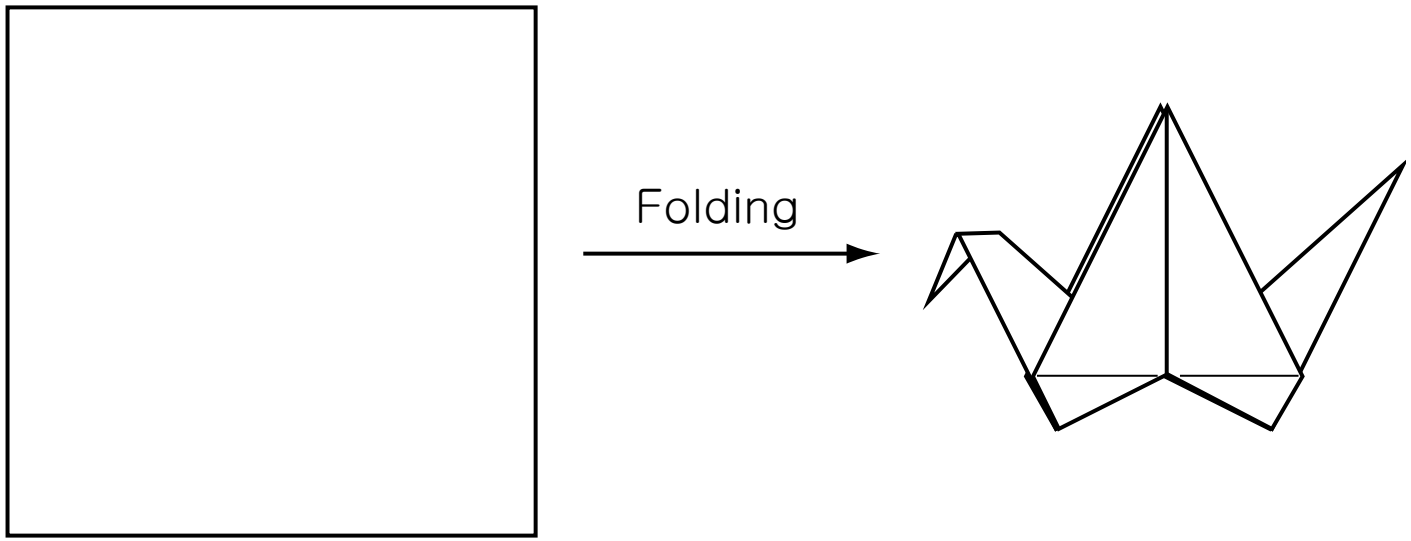


- Making three-dimensional structure from a paper by folding, without cutting or gluing the paper.

Goal of Origami



- Making three-dimensional structure from a paper by folding, without cutting or gluing the paper.
- However, sometimes the restriction is ignored to give a variety to the structure.

Definition of Origami

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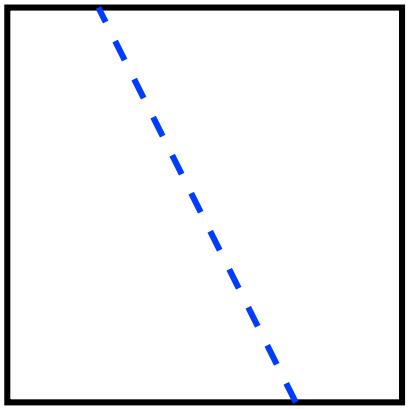
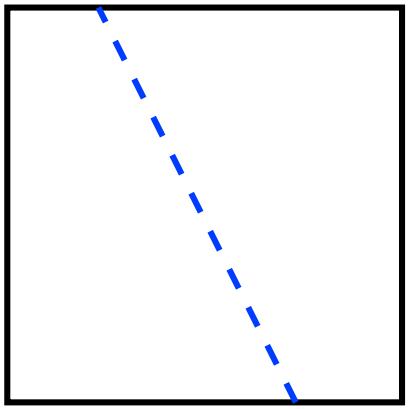


Line

Definition of Origami



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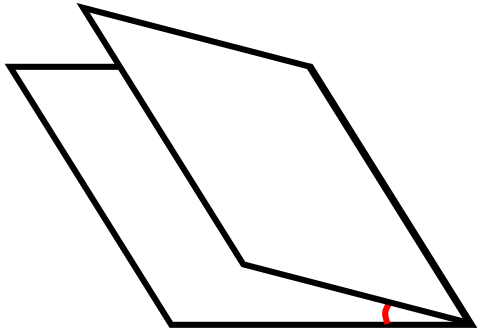
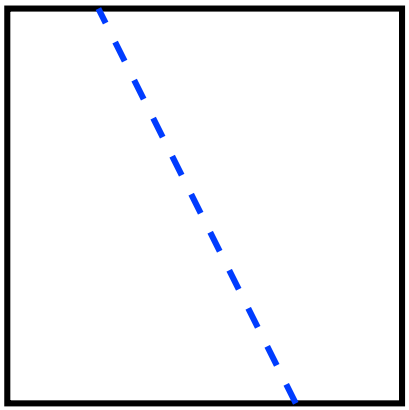
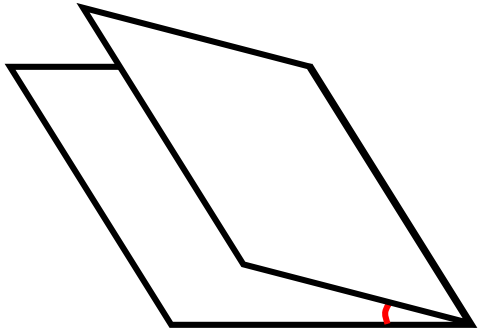


Angle

Definition of Origami



Line



Angle

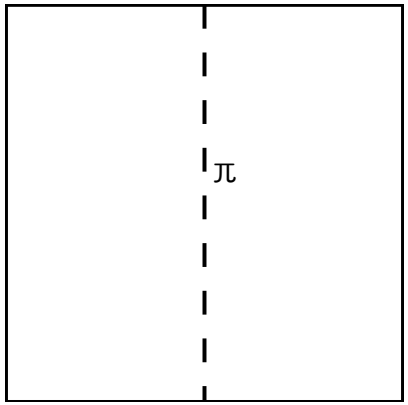
Definition (Origami : Simple Model)

Origami is a set of lines on finite plane with corresponding angle parameter. i.e.

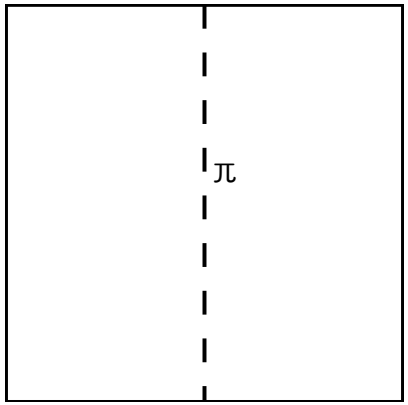
$$O : \{(\mathbf{l_1}, \theta_1), (\mathbf{l_2}, \theta_2), (\mathbf{l_3}, \theta_3), \cdots, (\mathbf{l_n}, \theta_n)\} \tag{1}$$

where $\mathbf{l_i}$ are lines and θ_i are corresponding angle parameter within $[-\pi, \pi]$.

Definition of Origami : Example



Definition of Origami : Example



$$O : \{(\mathbf{l_1}, \pi)\} \tag{2}$$

How to Analyze Origami?

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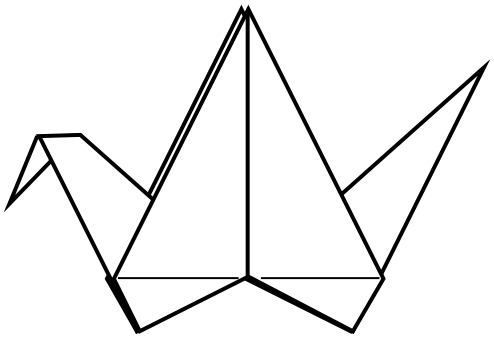

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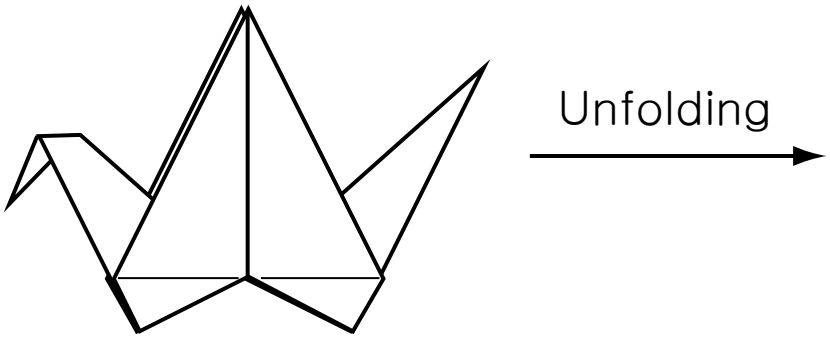

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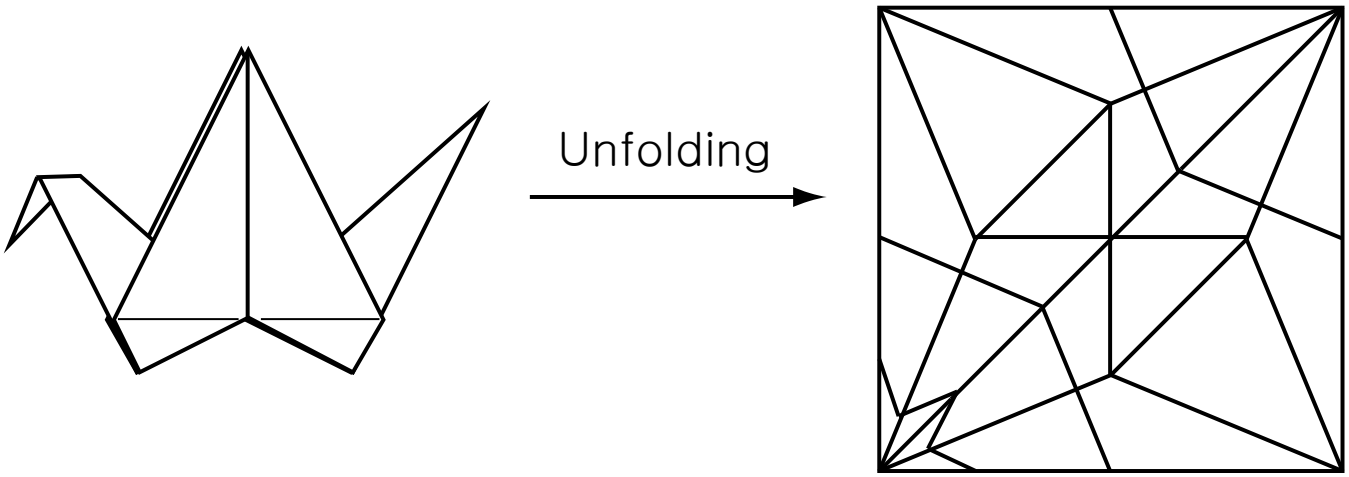
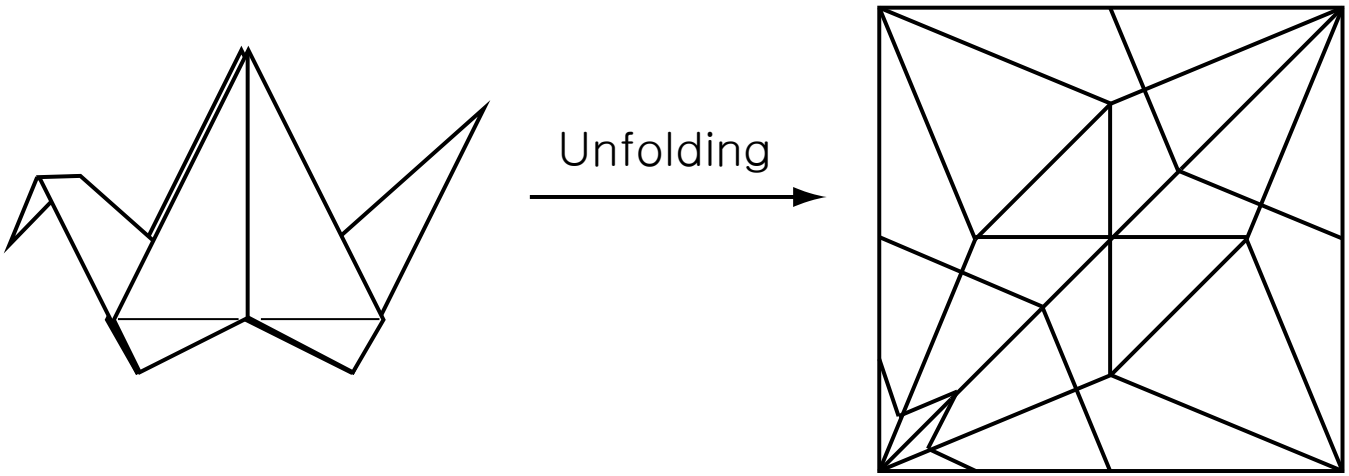


How to Analyze Origami?



- We call the last picture "crease pattern".

Definition of Flat Origami

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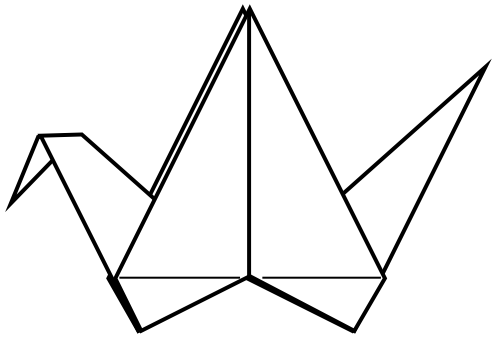
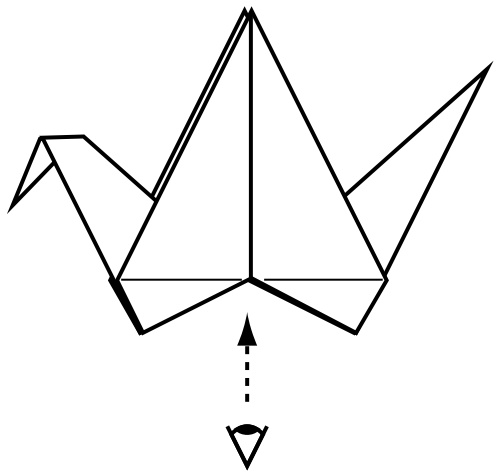


Definition of Flat Origami



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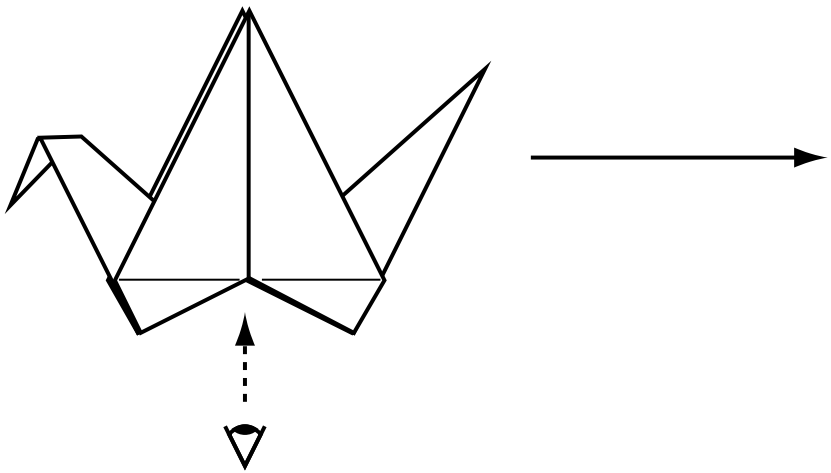
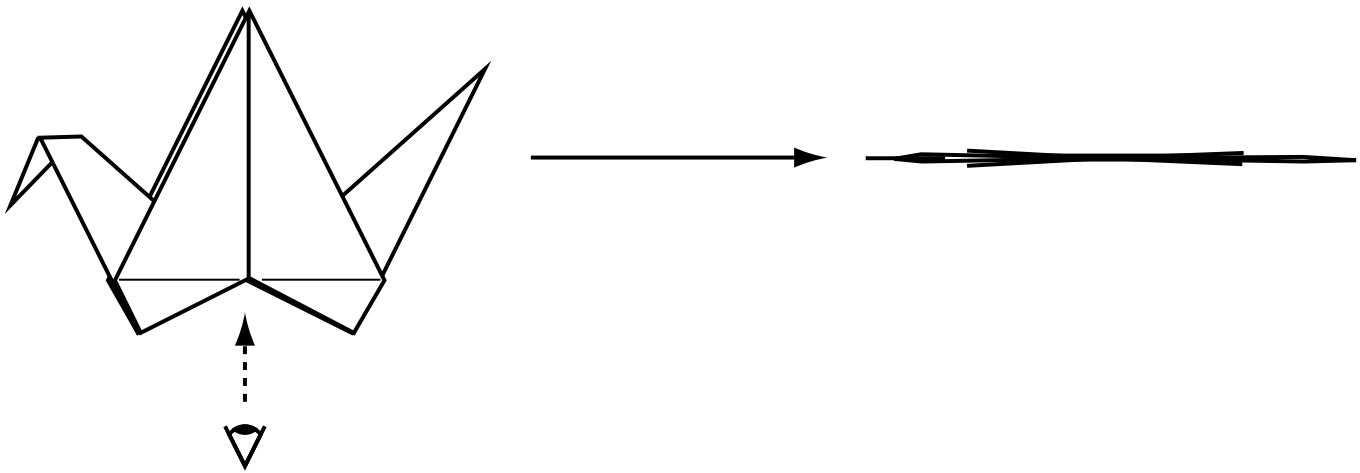


Definition of Flat Origami



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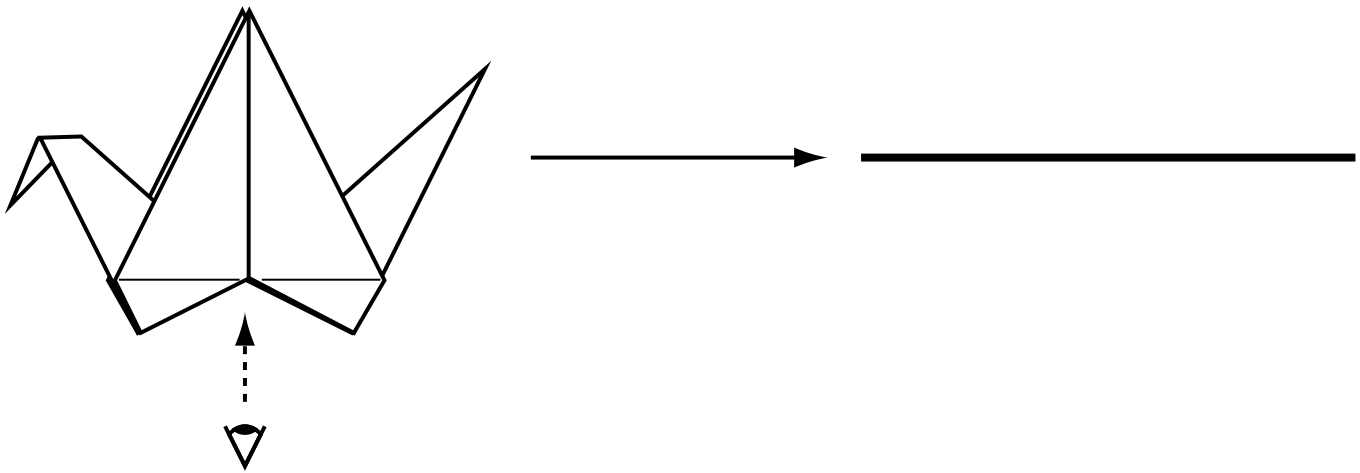
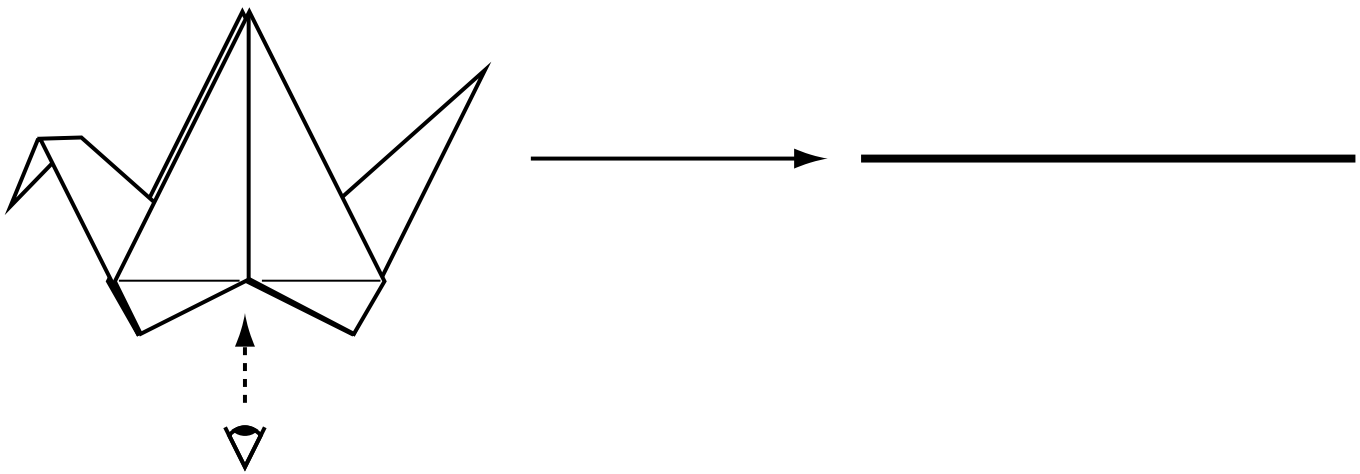


Definition of Flat Origami

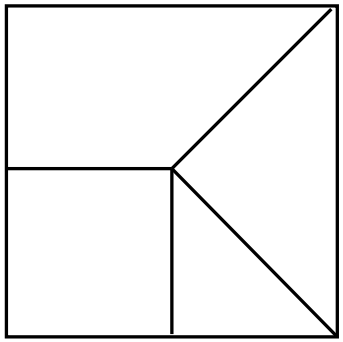
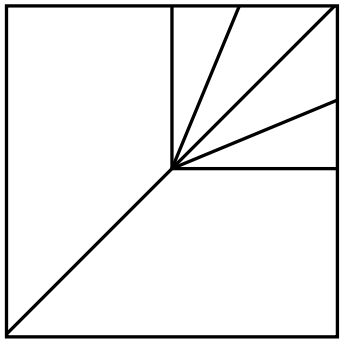
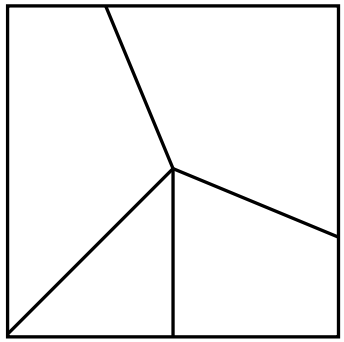
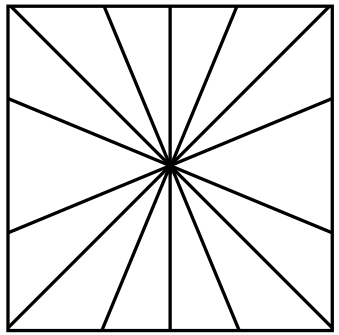


Definition (Flat Origami)

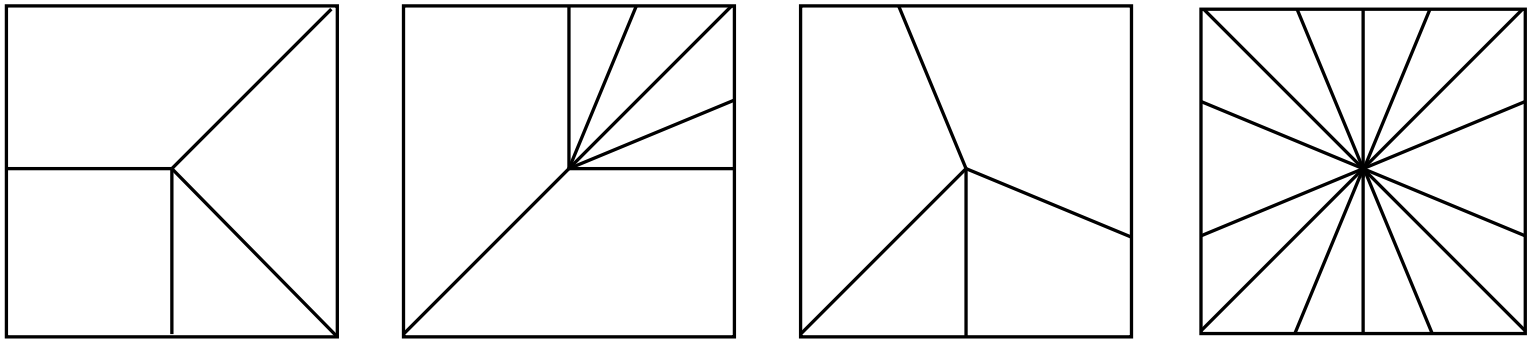
Flat origami is an origami with only angle parameters $-\pi$ and π.

Degree of a Vertex

Degree of a Vertex

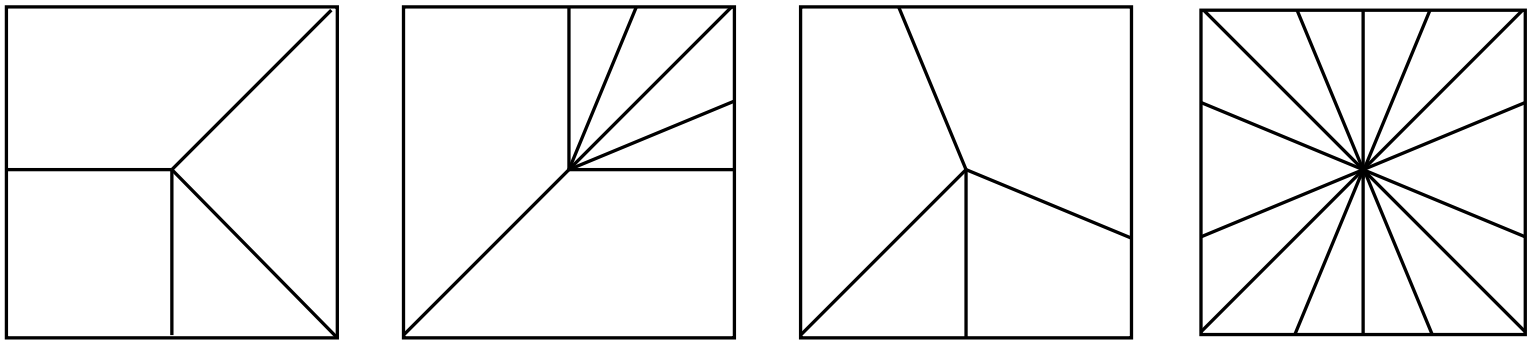





Degree of a Vertex



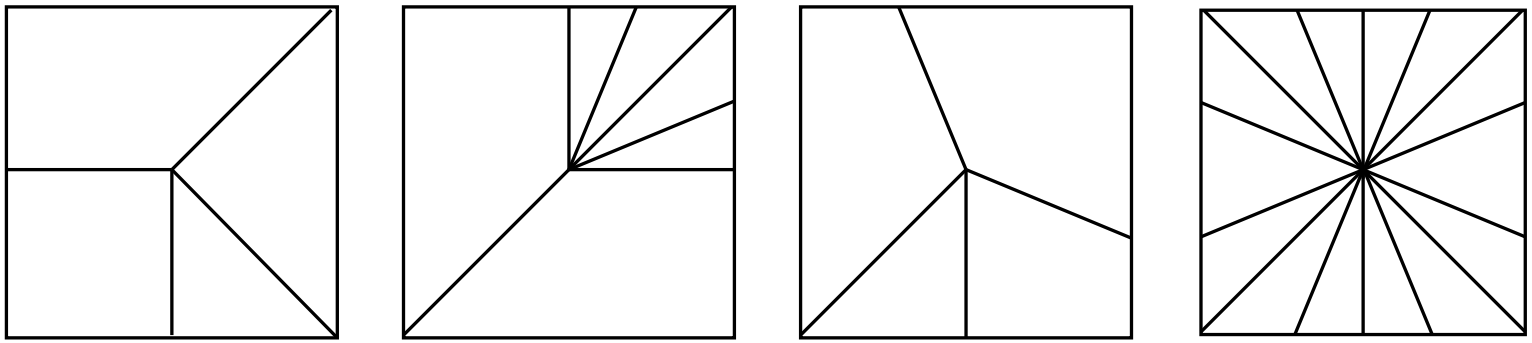
- Could you find something in common?

Degree of a Vertex



- Could you find something in common?
  - Degrees of each vertices are even.

Degree of a Vertex



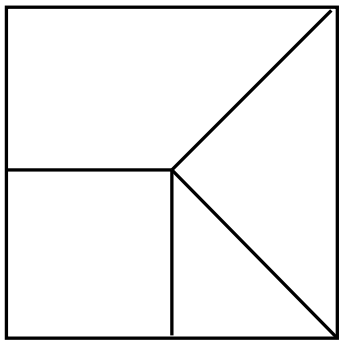
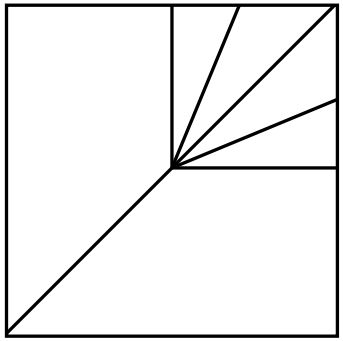
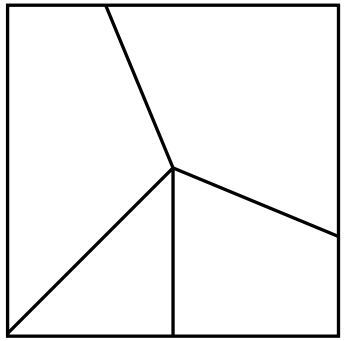
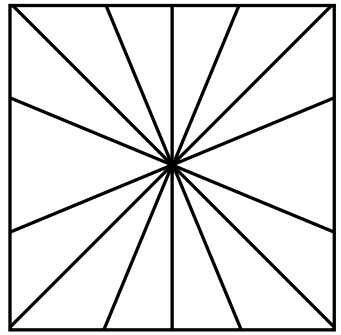
- Could you find something in common?
  - Degrees of each vertices are even.

Theorem

If vertex v is locally flat-foldable then,

$$deg(v) = 2n \qquad n = 1, 2, \cdots \tag{3}$$

Sum of Angles

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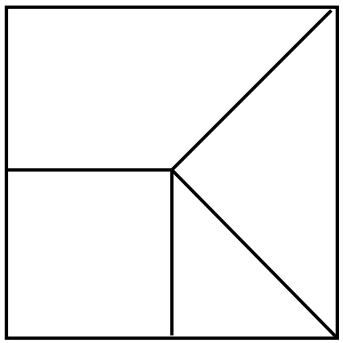
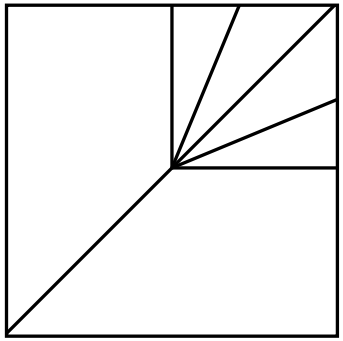
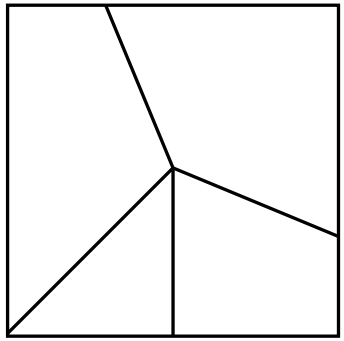
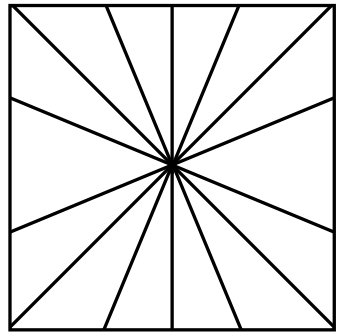
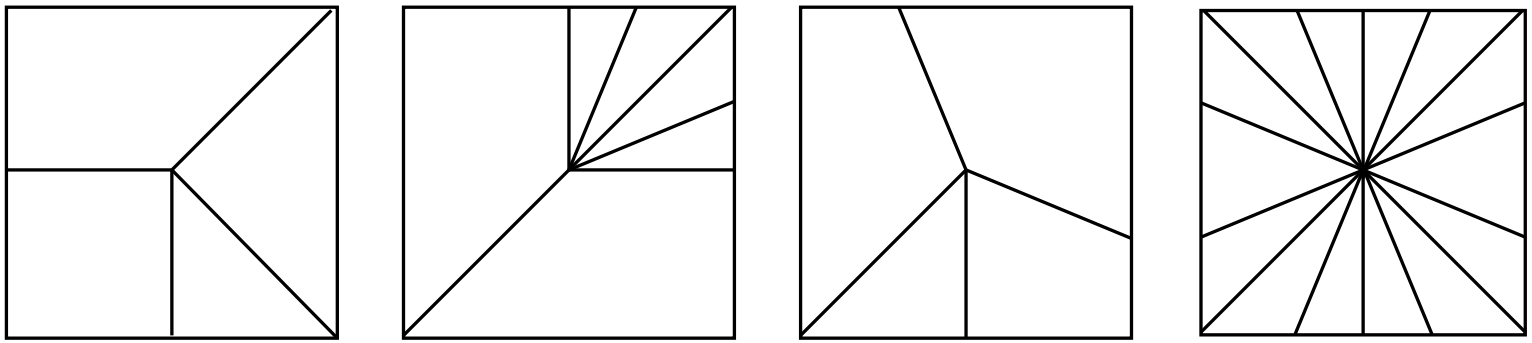





- Anything else?

Sum of Angles



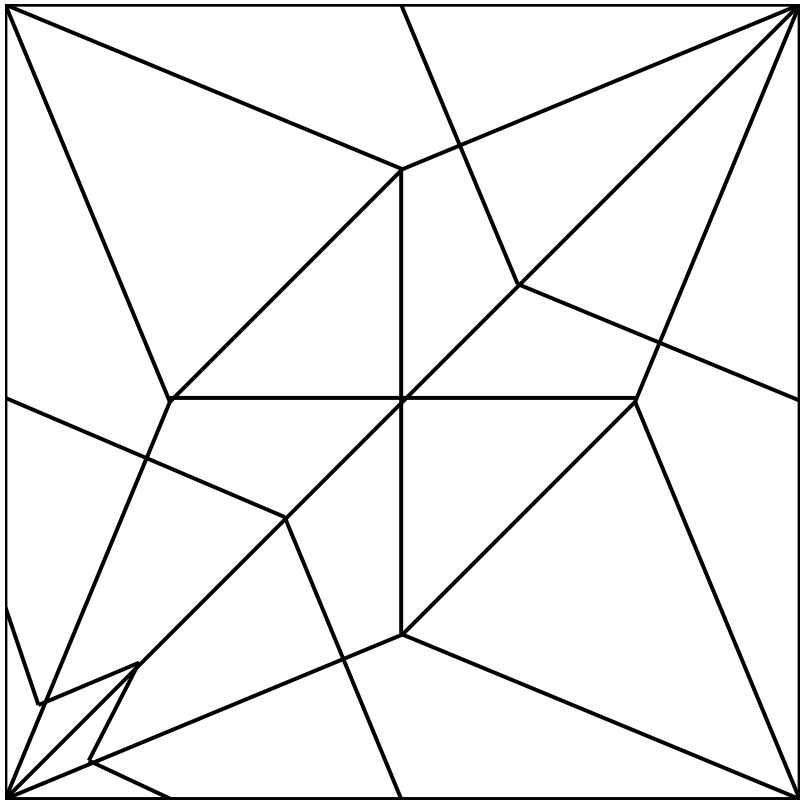
- Anything else?

Theorem (Kawasaki's Theorem)

Let v be a vertex of degree 2n in a single vertex fold and let $\alpha_1, \alpha_2, , \cdots, \alpha_{2n}$ *be the consecutive angles between the creases. Then v is a flat vertex fold if and only if*

$$\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \cdots - \alpha_{2n} = 0 \tag{4}$$

Example : Crease Pattern of Crane



Fold Number Difference

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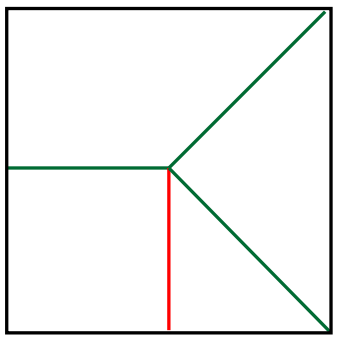
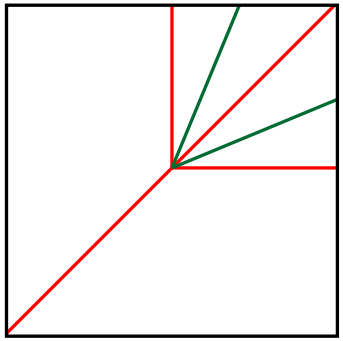
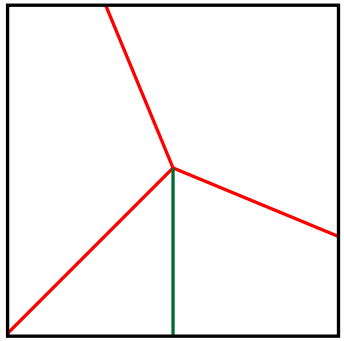
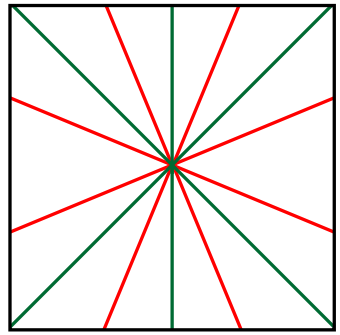

- Anything else?

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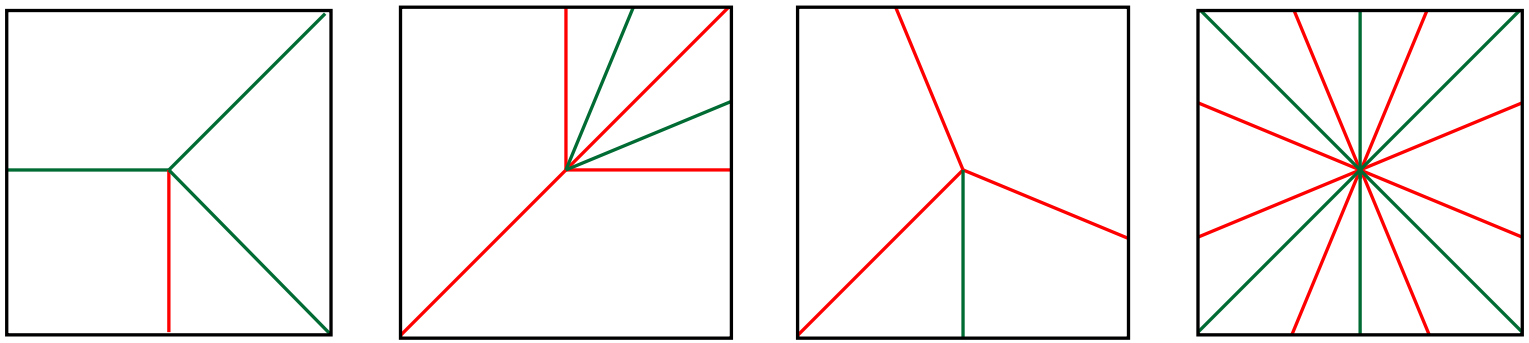


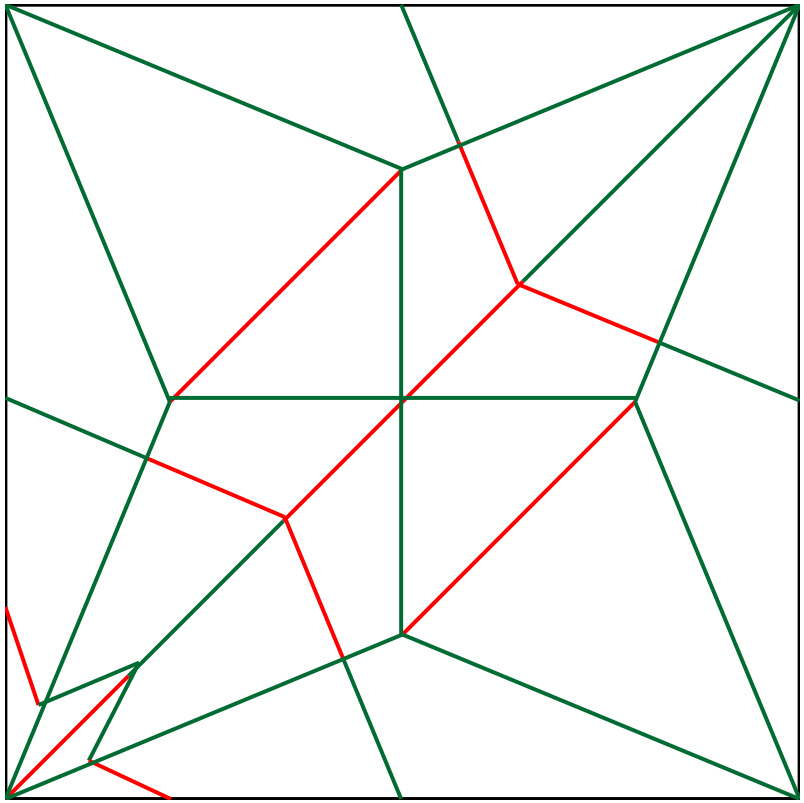
- Anything else?

Theorem (Maekawa's Theorem)

Let M be the number of mountain creases and V be the number of valley creases adjacent to a vertex in a single vertex fold. If a vertex v is locally flat-foldable then,

$$M - V = \pm 2 \tag{5}$$

Example : Crease Pattern of Crane



Quiz

Q : Can you find a non-flat foldable point in this crease pattern?

Quiz

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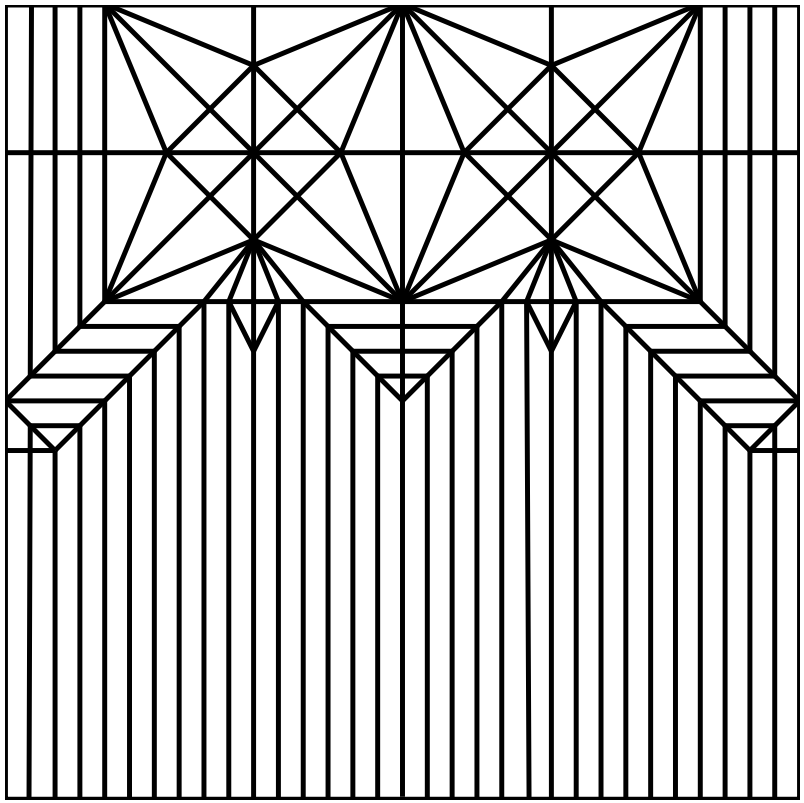
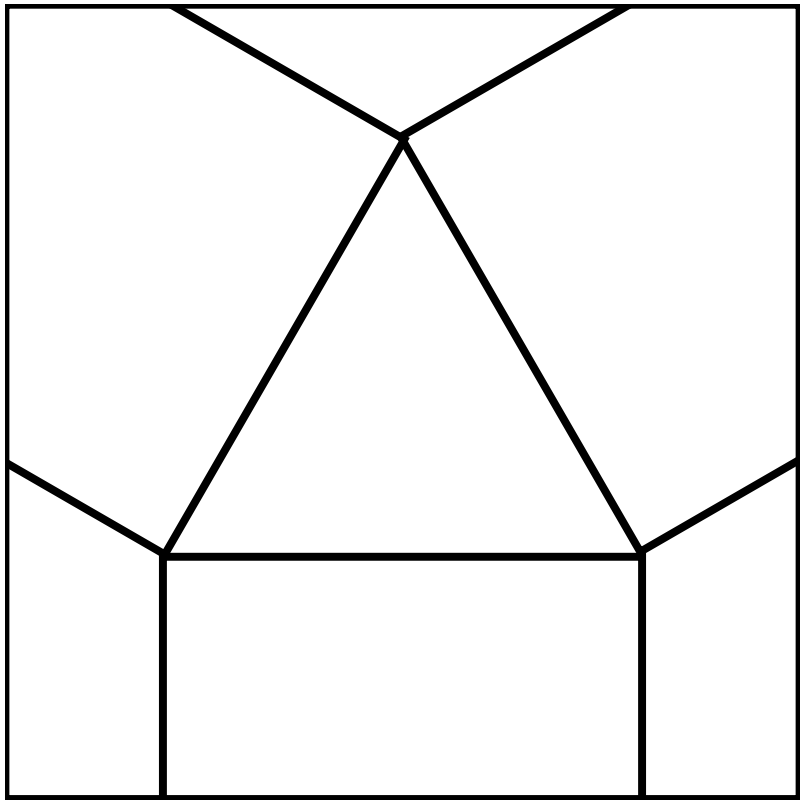


Locally Flat Foldable but not Globally



Global Flat Foldability

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- Because global flat foldability is depends on the whole structure not only the point, you shoud consider some area, not a point.

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- If you want to check it, see http://www.merrimack.edu/∼thull/papers/flatsurvey.pdf